CALABI–YAU MANIFOLDS WITH SPECIAL PROPERTIES

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Abstract. The object of study of this thesis are Calabi-Yau manifolds. These are important objects in algebraic geometry and have interesting properties. Together with irreducible holomorphic symplectic manifolds and tori, these are the building blocks for Kähler manifolds with zero first Chern class. The aim of the thesis is to study the Hodge numbers, mirror symmetry properties and the geometry of certain Calabi-Yau manifolds that can be obtained as quotients of product of lower dimensional Calabi-Yau manifolds, such as elliptic curves and K3 surfaces.

L'objet d'étude de cette thèse sont les variétés de Calabi-Yau. Ce sont des objets importants en géométrie algébrique et ils ont des propriétés intéressantes. Avec les variétés symplectiques holomorphes et les tores, ce sont les pierres pour construire les variétés kähleriénnes avec première classe de Chern nulle. Le but de la thèse est d'étudier les nombres de Hodge, les propriétés miroirs et la géométrie de certaines variétés de Calabi-Yau que l'on obtient comme quotients de produits de variétés de Calabi-Yau de dimension inférieure, comme les courbes elliptiques et les surfaces K3.

1. Description of the PhD topic

We consider the product of an elliptic curve E and a K3 surface S, both with a non-symplectic involution, that we denote respectively by ι and j. It was shown by Borcea and Voisin, that there exists a crepant resolution of the quotient $E\times S/\langle\iota\times j\rangle$ that is a Calabi-Yau threefold. Voisin computed the Hodge numbers of the threefold by using the properties of the non-symplectic involution on the K3 surface and she determined mirror families. Since the papers of Borcea and Voisin a lot of work has been done around the subject, in particular the construction has been generalized in higher dimension. The aim of the thesis is to study properties of the Calabi-Yau manifolds obtained as generalizations of the Borcea-Voisin construction. In particular, one should investigate mirror symmetry properties of the variety in relation with the lattice mirror symmetry for K3 surfaces. This has been done in higher dimension only for Calabi-Yau fourfolds by Dillies. It would be interesting to understand how far one can use Borcea-Voisin generalized construction to get mirror families of Calabi-Yau manifolds. One method to attack the investigation is the use of the orbifold cohomology introduced by Chen-Ruan.

It would also be interesting to study automorphisms of the Calabi-Yau varieties produced in this way (one should use recent works of Favale, Oguiso) and special fibrations (e.g. elliptic fibrations) on these varieties (works of Garbagnati and Cattaneo).

Finally another possible direction of the thesis is to classify all the quotients of a product a of a K3 surface and an elliptic curve by a finite group of automorphism G

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that acts as a translation on the elliptic curve and as a symplectic automorphisms group of finite order on the K3 surface. The aim is to describe higher dimensional generalizations of bielliptic surfaces.

Useful papers, other than the papers mentioned above, are papers by Cynk-Hulek, Dillies, Bini and Artebani-Boissière-Sarti.

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